12/21/16

"Some are born to greatness, some achieve greatness, and some have greatness thrust upon them"
- William Shakespeare

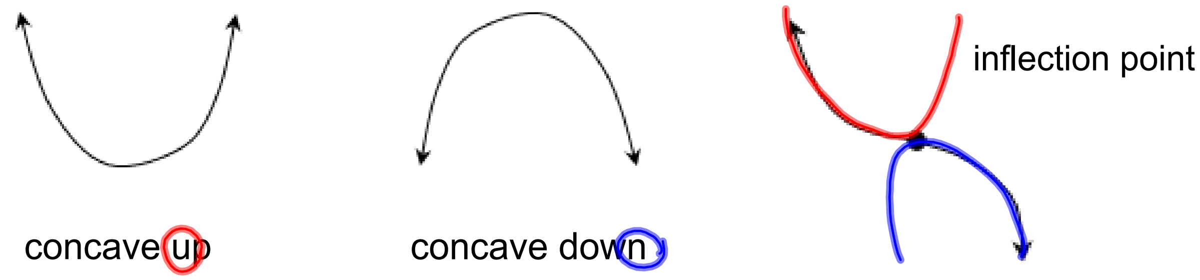
HW: "Concavity" #1-4

AIM: How do we determine the concavity of a function?

Warm Up:

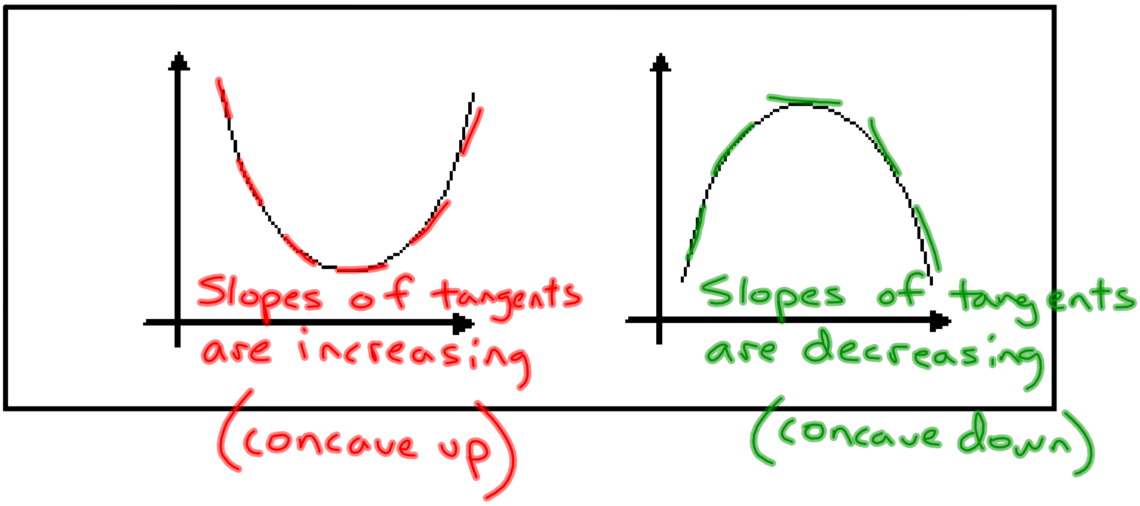
CONCAVITY AND
THE SECOND DERIVATIVE TEST

The first derivative describes the direction of the function. The second derivative describes the concavity of the original function. Concavity describes the direction of the curve, how it bends...



Just like direction, concavity of a curve can change, too. The points of change are called ***inflection points***.

CONCAVITY EXPLORATION: Draw small tangent lines at points along the curves below. What do you notice about the slopes of the tangent lines (the derivatives) as you move from left to right at these points?



TEST FOR CONCAVITY

If $f''(x) > 0$, then graph of *f* is concave up.

If $f''(x) < 0$, then graph of *f* is concave down.

SUMMARY OF FIRST AND SECOND DERIVATIVE TESTS			
	$f'(x) > 0$	$f'(x) < 0$	$f'(x) = 0$
$f''(x) > 0$	Increasing Concave Up	Decreasing Concave Up	Relative Minimum Concave Up
$f''(x) < 0$	Increasing Concave Down	Decreasing Concave Down	Max Relative ~~Minimum~~ Concave Down
$f''(x) = 0$	Increasing Inflection Point	Decreasing Inflection Point	Function is smooth, level” possible inflection point

$$f(x) = \frac{x}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$2x^2$

$$f''(x) = \frac{(x^2+1)^{\cancel{2}}(-2x) - (1-x^2)(2\cancel{(x^2+1)}(2x))}{(x^2+1)^{3}}$$

$(1-x^2)\,4x$

$$-2x^3-2x-(4x-4x^3)$$

$$\frac{-2x^3-2x-4x+4x^3}{(x^2+1)^3}$$

$$\frac{2x^3-6x}{(x^2+1)^3}$$

EX #1: Given $f(x)=\frac{1}{3}x^3-x$, determine the open intervals on which the graph is concave upward or downward.

STEP 1: Find the first derivative. $f'(x)$

$f'(x)=x^2-1$

STEP2: Find the second derivative. $f''(x)$

$f''(x)=2x$

STEP 3: Find the critical values. $f''(x)=0$

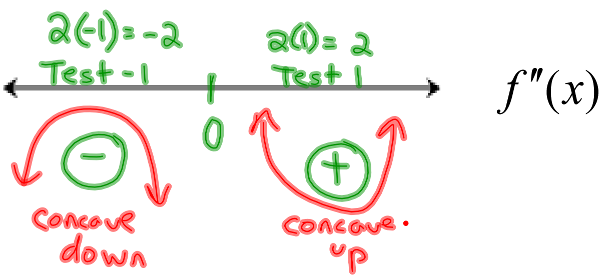
$2x=0$
$x=0$

Make a sign chart for $f''(x)$

Test #

Critical #

Sign



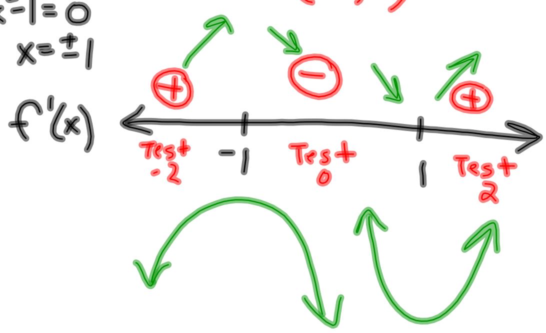
STEP 4: Find intervals for increasing/decreasing

first derivative

Increases: $(-\infty,-1)\cup(1,\infty)$

Decreases: $(-1,1)$

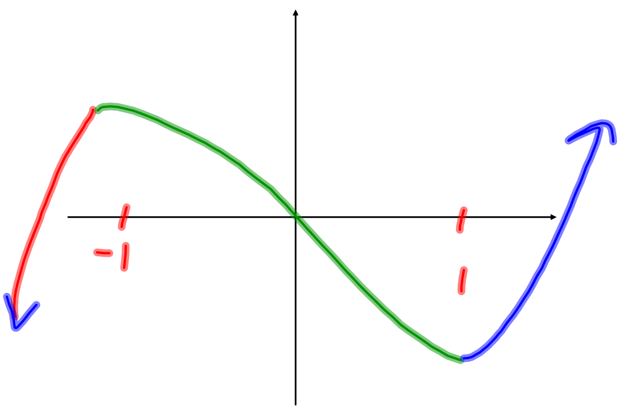
$x^2-1=0$
$x=\pm 1$

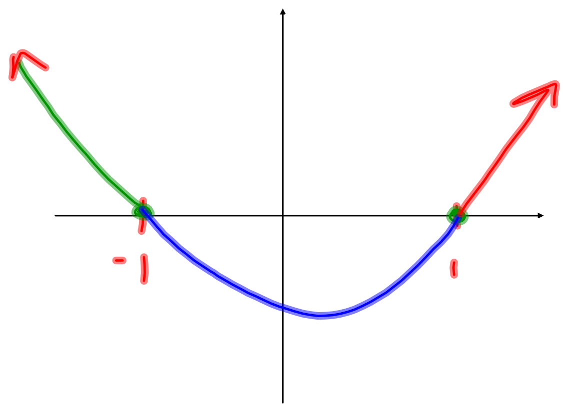


EX #2: Graphs and Derivatives

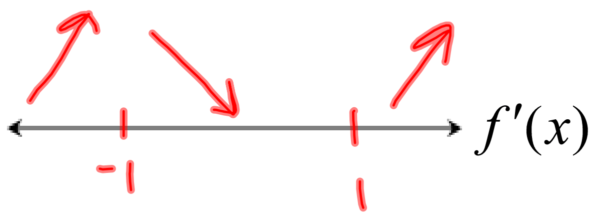
The concavity $\left(f''(x)\right)$ and direction $\left(f'(x)\right)$ of the function $\left(f(x)\right)$ is related to the slope of the derivative.

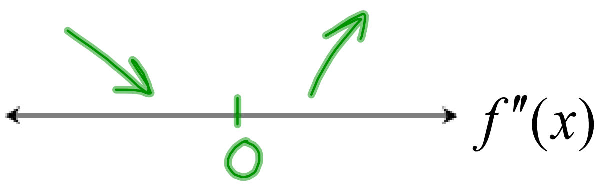
$$f\left(x\right)=\frac{1}{3}x^{3}-x \qquad f'\left(x\right)=x^{2}-1$$





Summary



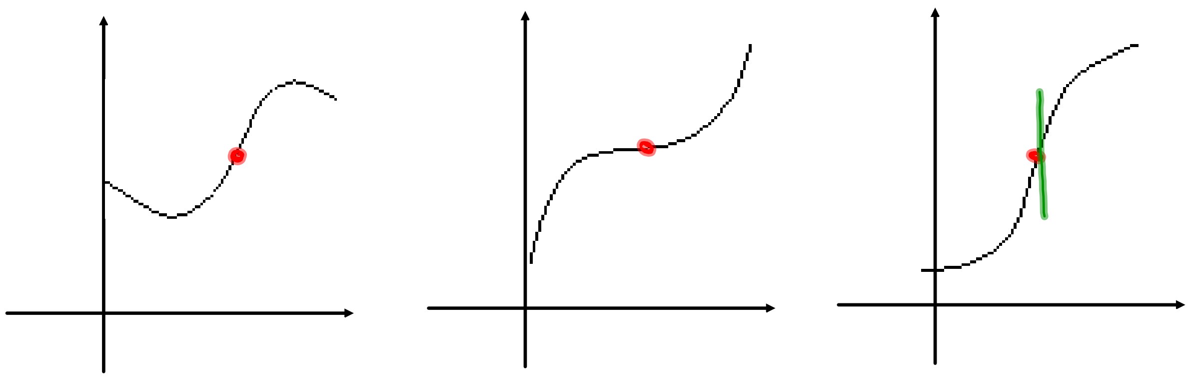


POI

POINTS OF INFLECTION:

The concavity of *f* changes at a point of inflection.

Where $f''(x) = 0$ or $f''(x) = does\ not\ exist$



EX #3: Determine any points of inflection and discuss concavity of the graph of $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0 \quad | \quad x = 3$

$f''(x) = 12x^2 - 24x$

$12x(x-2)$

$x = 0 \quad | \quad x = 2$

Test #	-1		1		3
Critical #		0		2	
Sign $f''(x) =$	$+$		$-$		$+$

Concave Up: $(-\infty, 0) \cup (2, \infty)$

Concave Down: $(0, 2)$

EX #4: Use the Second Derivative Test to determine the relative extrema for $f\left(x\right)=-3x^5+5x^3$

Step 1: Find critical numbers where $f'(x)=0$

$f'(x)=-15x^4+15x^2$

$-15x^4+15x^2=0$

$-15x^2(x^2-1)=0$

$-15x^2(x+1)(x-1)=0$

$x=0$ | $x=-1$ | $x=1$

Possible Max/Min places

Step 2: Find $f''(x)$

$f''(x)=-60x^3+30x$

Step 3: Find sign of $f''(x)$ for each critical number.

$f''(0)=-60(0)^3+30(0)$

$f''(0)=0$

$f''(-1)=-60(-1)^3+30(-1)$

$f''(-1)=30$

$x=-1$ Min

$f''(1)=-60(1)^3+30(1)$

$f''(1)=-30$

$x=1$ Max

Critical Point	$(0,0)$	$(-1,-2)$	$(1,2)$
Sign of $f''(x)$	$f''(0)=0$	$f''(-1)>0$	$f''(1)<0$
Conculsion	Neither	Min	Max

EX #5: Use First and Second Derivative Tests to determine behavior of *f* and graph.

Given: $f(x) = 3x^4 - 4x^3 + 6$

1. $f'(x) = 0$

$f'(x) = 12x^3 - 12x^2$

$12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$x = 0$ $x = 1$

2. Critical points

$(0,6)$ $(1,5)$ Min

3. First Derivative Test

4. $f''(x) = 0$

$f''(x) = 36x^2 - 24x$

$36x^2 - 24x = 0$

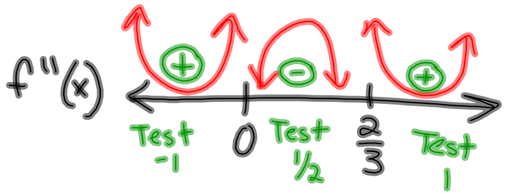
$12x(3x-2) = 0$

$x = 0$ $x = \frac{2}{3}$

5. Points of Inflection

$(0,6)$ $(\frac{2}{3}, 5.4)$

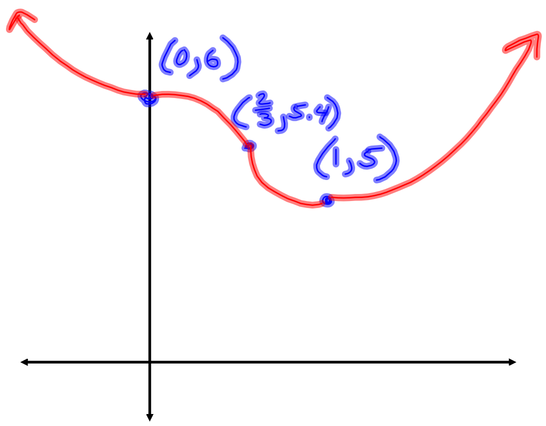
6. Second Derivative Test



7. Summarize

Critical Points (*c*)	*f'*(*c*)	*f''*(*c*)	~~Conclusion~~ Conclusion	Point of Inflection
(0,6)	0	0	Not Max/Min	✓
(1,5)	0	+12	Min	✗

8. Graph



Function: Points

First Derivative: Increase/Decrease
Set = 0 to find possible Max/min

Second Derivative: Concave Up / Down
Set = 0 to find Points of inflection